∗ PROBLEM 11: A NEW THEORY OF THE WEAK INTERACTIONS *(40 points)*

This problem was Problem 3, Quiz 3, 2009.

Suppose a New Theory of the Weak Interactions (NTWI) was proposed, which differs from the standard theory in two ways. First, the NTWI predicts that the weak interactions are somewhat weaker than in the standard model. In addition, the theory implies the existence of new spin-$\frac{1}{2}$ particles (fermions) called the R^+ and R^-, with a rest energy of 50 MeV (where 1 MeV $= 10^6$ eV). This problem will deal with the cosmological consequences of such a theory.

The NTWI will predict that the neutrinos in the early universe will decouple at a higher temperature than in the standard model. Suppose that this decoupling takes place at $kT \approx 200$ MeV. This means that when the neutrinos cease to be thermally coupled to the rest of matter, the hot soup of particles would contain not only photons, neutrinos, and e^+-e^- pairs, but also μ^+, μ^-, π^+, π^-, and π^0 particles, along with the R^+-R^- pairs. (The muon is a particle which behaves almost identically to an electron, except that its rest energy is 106 MeV. The pions are the lightest of the mesons, with zero angular momentum and rest energies of 135 MeV and 140 MeV for the neutral and charged pions, respectively. The π^+ and π^- are antiparticles of each other, and the π^0 is its own antiparticle. Zero angular momentum implies a single spin state.) You may assume that the universe is flat.

(a) *(10 points)* According to the standard particle physics model, what is the mass density ρ of the universe when $kT \approx 200$ MeV? What is the value of ρ at this temperature, according to NTWI? Use either g/cm^3 or kg/m^3. (If you wish, you can save time by not carrying out the arithmetic. If you do this, however, you should give the answer in "calculator-ready" form, by which I mean an expression involving pure numbers (no units), with any necessary conversion factors included, and with the units of the answer specified at the end. For example, if asked how far light travels in 5 minutes, you could answer $2.998 \times 10^8 \times 5 \times 60$ m.)

(b) *(10 points)* According to the standard model, the temperature today of the thermal neutrino background should be $(4/11)^{1/3} T_\gamma$, where T_γ is the temperature of the thermal photon background. What does the NTWI predict for the temperature of the thermal neutrino background?

(c) *(10 points)* According to the standard model, what is the ratio today of the number density of thermal neutrinos to the number density of thermal photons? What is this ratio according to NTWI?

(d) *(10 points)* Since the reactions which interchange protons and neutrons involve neutrinos, these reactions "freeze out" at roughly the same time as the neutrinos decouple. At later times the only reaction which effectively converts neutrons to protons is the free decay of the neutron. Despite the fact that neutron decay is a weak interaction, we will assume that it occurs with the usual 15 minute mean lifetime. Would the helium abundance predicted by the NTWI be higher or lower than the prediction of the standard model? To within 5 or 10%, what would the NTWI predict for the percent abundance (by weight) of helium in the universe? (As in part (a), you can either carry out the arithmetic, or leave the answer in calculator-ready form.)

Useful information: The proton and neutron rest energies are given by $m_p c^2 = 938.27$ MeV and $m_n c^2 = 939.57$ MeV, with $(m_n - m_p)c^2 = 1.29$ MeV. The mean lifetime for the neutron decay, $n \to p + e^- + \bar{\nu}_e$, is given by $\tau = 886$ s.

(a) Mass density of the universe (both models)

(a) What is the mass density ρ of the universe when $kT = 200$ MeV according to both the standard model and NTWI.

thermal equilibrium

Recall that the mass density can be calculated using

$$\rho = \frac{u}{c^2} = \underbrace{g_{TOT}}_{\text{we need this}} \underbrace{\frac{\pi^2}{30}\frac{(kT)^4}{\hbar^3 c^5}}_{\text{known}}$$

SM:

$$\left.\begin{aligned}
\gamma &: g = 2 \\
e^+e^- &: g = 2\times 2\times \tfrac{7}{8} = 7/2 \\
\nu_e, \nu_\mu, \nu_\tau &: g = 3\times 2\times \tfrac{7}{8} = 21/4 \\
\mu^+\mu^- &: g = 2\times 2\times \tfrac{7}{8} = 7/2 \\
\pi^+\pi^-\pi^0 &: g = 3
\end{aligned}\right\} \Longrightarrow g_{TOT} = 2 + \frac{7}{2} + \frac{21}{4} + \frac{7}{2} + 3 = 5 + 7 + \frac{21}{4} = 69/4 = 17\frac{1}{4}$$

$\Longrightarrow$ In the SM,

$$\rho = \frac{69}{4}\cdot\frac{\pi^2}{30}\frac{(kT)^4}{\hbar^3 c^5}$$

$$= \frac{69}{4}\cdot\frac{\pi^2}{30}\cdot\frac{\left(200\times 10^6\,\text{eV}\times\frac{1.602\times 10^{-19}\,\text{J}}{1\,\text{eV}}\right)^4}{(1.055\times 10^{-34}\,\text{J s})^3\cdot(2.998\times 10^8\,\text{m/s})^5}$$

$$= \underbrace{\frac{69\cdot\pi^2}{120}\frac{(3.204\times 10^{-11})^4}{(1.055\times 2.988\times 10^{-26})^3}\frac{1}{(2.998\times 10^8)^2}}\cdot\frac{\text{J}^4}{\text{J}^3\,\text{s}^3\,\frac{\text{m}^5}{\text{s}^5}}$$

$$= (\#)\cdot\frac{\text{J}}{(\text{m}^2/\text{s}^2)\cdot\text{m}^3} = (\#)\cdot\frac{\text{kg}}{\text{m}^3}$$

Calculator $\longrightarrow$ $= 2.10\times 10^{18}\ \text{kg/m}^3$

In NTWI,

all we need is the additional contribution of $R^+ - R^-$: $g = \underset{\text{spin states}}{2}\times 2\times \underset{\text{Fermion}}{7/8}$ (particle/antiparticle)

$\Longrightarrow$ An extra $7/2 = 14/4$

$$\Longrightarrow g_{TOT} = \frac{69}{4} + \frac{14}{4} = \frac{83}{4}$$

$$\Longrightarrow \rho_{NTWI}^{kT=200\,\text{MeV}} = \frac{83}{4}\cdot\frac{\pi^2}{30}\frac{(kT)^4}{\hbar^3 c^5} = \frac{83}{69}\rho_{SM}^{kT=200\,\text{MeV}} = 2.53\times 10^{18}\ \text{kg/m}^3$$

(b) The temperature of the neutrino background is obtained by noting that the entropy of non-interacting species is separately conserved in thermal equilibrium.

$\Longrightarrow$ After neutrinos decouple, $a^3 S_\nu$ and $a^3 S_{other}$ are conserved

neutrino entropy density

The entropy density can be expressed as $s = A g T^3$, where A is a constant.
Just after neutrinos decouple, we have

NTWI: $s_\nu = \underline{\frac{21}{4}} A T^3$, $s_{other} = \underline{\left(\frac{83}{4} - \frac{21}{4}\right)} A T^3 = \underline{\frac{62}{4}} A T^3$

After all the massive particles annihilate, $s_{other} = s_{photons} = 2 A T_\gamma^3$

$$\nu: \Rightarrow \frac{21}{4} A (aT)^3 = (a^3 s_\nu)_{\substack{\text{neutrino}\\\text{decoupling}}} = (a^3 s_\nu)_{CNB} = \frac{21}{4} A (a T_\nu)^3$$

$$\gamma: \Rightarrow \frac{31}{2} A (aT)^3 = (a^3 s_{other})_{\substack{\text{neutrino}\\\text{decoupling}}} = (a^3 s_{other})_{CMB} = 2A (a T_\gamma)^3$$

procedure same as problem 19

Dividing, $\frac{1}{2} \cdot \frac{21}{31} = \frac{21}{8} \left(\frac{T_\nu}{T_\gamma}\right)^3 \Rightarrow \underline{T_\nu = T_\gamma \left(\frac{4}{31}\right)^{1/3}}$ //

(c) For number densities, one can write $\boxed{n = B g^* T^3}$

some constant

$\underline{g^*_\nu} = \frac{3}{4} \times 3 \times 2 = \frac{9}{2}$, $\underline{g^*_\gamma} = 2$

NTWI: $\frac{n_\nu}{n_\gamma} = \frac{B \cdot 9/2 \; T_\nu^3}{B \cdot 2 \; T_\gamma^3} = \frac{9}{4} \cdot \left(\frac{T_\nu}{T_\gamma}\right)^3 = \frac{9}{4} \cdot \frac{4}{31} = \boxed{\frac{9}{31}}$ //

SM: $\frac{n_\nu}{n_\gamma} = \frac{9}{4} \left(\frac{T_\nu}{T_\gamma}\right)^3 = \frac{9}{4} \cdot \frac{4}{11} = \boxed{\frac{9}{11}}$ //

(d) What happens with Helium abundance in NTWI?

At $\underline{kT = 200 \text{ MeV}}$, $\frac{n_n}{n_p} = e^{-(m_n - m_p)c^2/kT} = e^{-1.29 \text{ MeV}/200 \text{ MeV}}$

≈ 1 //

This is much larger than the SM ratio, because kT at decoupling is larger.
Since free neutron decay is a rather slow process, we conclude that NTWI predicts a higher Helium abundance.

Now to quantify this:

As a first approximation, we can take the time it takes in the SM to get to nucleosynthesis (Weinberg gives ~ 225 s, Ryden ~ 200 s), and say that the fraction of neutrons that do not decay

$Y = e^{-t_{nuc}/\tau_d}$, where $\tau_d \sim 890$ s is the free neutron decay lifetime

$$\Rightarrow \quad Y = e^{-200/890} \sim 0.8 .$$

Since at T= 200 MeV we essentially had $n_n \approx n_p$, if it weren't for free decay, all neutrons/protons could be bound into Helium. If we bind all remaining neutrons into Helium, the weight fraction abundance of Helium in the universe is simply $Y \sim 0.8$.

One can improve this estimate by recalculating t_{nuc} using the g_{eff} number of degrees of freedom of NTWI.

One obtains $t_{nuc}^{NTWI} = \sqrt{\dfrac{g_{eff}^{SM}}{g_{eff}^{NTWI}}}\, t_{nuc}^{SM}$ ← This comes from $t \propto \dfrac{1}{g_{eff}^{1/2} T^2}$ in a radiation-dominated universe.

$$\Rightarrow \quad t_{nuc}^{NTWI} = 1.20\, t_{nuc}^{SM}$$

$$\Rightarrow \quad Y \sim 0.76$$